NOTES ON THE APLICATION OF SPRING CONSTANT AND SOIL STRUCTURE INTERACTION PROBLEM

GOUW Tjie-Liong

Senior Geotechnical Consultant
Green Garden Blok I-9 No. 28 A - Jakarta 11520 – INDONESIA
Email: *geotech@cbn.net.id*

ABSTRACT

Despite the advancement of the computer technology and engineering software, which allows soil structure interaction to be analyzed economically, the concept of spring constant is still widely used in analyzing raft and pile raft foundation. This paper first reviews the spring constant concept, its limitation and the misused of the value in evaluating building foundation. Finally, a case study on a building, soil and tunnels interaction is given as an example of solving a soil structure interaction problem by using geotechnical finite element software.

KEYWORDS

Spring Constant, Coefficient of Subgrade Reaction, PLAXIS software

1. INTRODUCTION

Recently, over the period of December 2000 to February 2001, there is an intensive discussion about "spring constant" in the World Wide Web (see: http://www.indoconstruction.com), i.e., the Internet version of the "Indo-construction" magazine. The points of the discussions are on the limitation of the theory, the proper estimation of the spring constant magnitude and the soil structure interaction.

Coincidentally, in 1988, the author had an interesting experience on the application of the spring constant for a design of a Mass Rapid Transit railway station in an oversea project. The overview of soil condition on that particular site is as shown in Fig. 1 below.

At this project the spring constant concept was adopted for designing the station raft foundation. The structural engineer asked for the magnitude of the spring constant from a young geotechnical engineer, who then gave a coefficient of subgrade reaction (in kN/m^3) derived from a plate-loading test. This parameter was later converted into a foundation coefficient of subgrade reaction, k_s, by using the following equation:

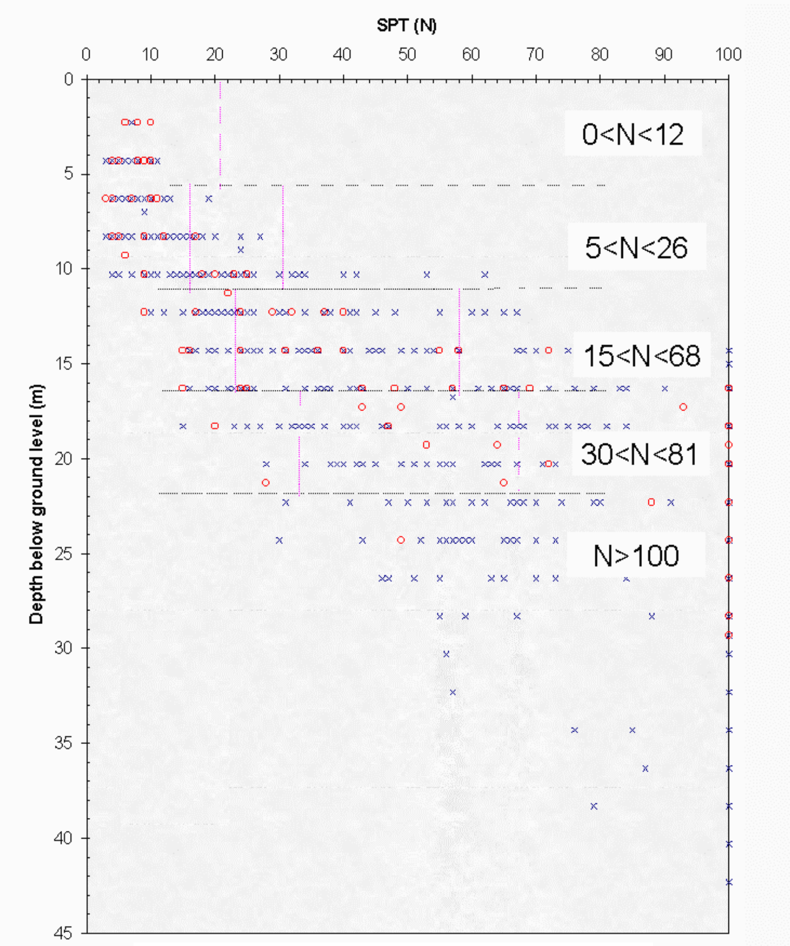

Fig. 1 - SPT vs Depth

$$k_s = k\left[\frac{B+0.3}{2B}\right]^2 \qquad \text{……. (1)}$$

where B is the width of the raft foundation.

This last parameter was then applied as a spring constant by multiplying it with the unit area under the raft foundation (the unit dimension became kN/m). A certified Professional Engineer then approved the outcome of the raft foundation design for construction.

Without prejudice to blame others, it is obviously a mistake! Why it is so? For B greater than 0.3 m, equation 1 clearly shows that the greater the value of B the smaller the value of k_s. While it is structurally correct that the wider the foundation the more flexible the foundation is. It does not equally right for the foundation soil. The engineers had missed the fact that the soil at that area was far from homogeneous.

The soil condition shows that, within the influence of the raft foundation, the deeper the foundation soils the harder they are. This means the deeper soils have greater rigidity as compared to the layer right below the raft foundation (note: the width of the raft is around 35 m).

The inappropriate spring constant led to an excessive settlement of the raft. As a result, in order to reduce the settlement, the center of the raft was strengthened with more than 20 number of bored piles. Upon reviewing the design, the author proved that the bored piles were excessive and unnecessary. However, by the time it was found, it was too late.

The above case shows the application of spring constant without considering the characteristics and the behavior of the underlying soils. And it is also an example of the existence of ignorance, gaps and weakness in the relation among the structural and geotechnical engineers. This papers tries to elaborate the underlying principle the spring constant theory, its limitation and the application of specially made geotechnical software to solve the problem of soil structure interaction.

2. SPRING CONSTANT - THE THEORETICAL BACKGROUND AND THE LIMITATION

“What is the spring constant at this particular site?” or “What is the modulus of subgrade reaction at this location?” is a common question asked by a structural engineer to a geotechnical engineer. It is a straightforward question. Unfortunately, it has no direct, let alone a simple answer.

The concept of *spring constant* was first introduced by Wrinkler in 1867. He modeled flexible foundation, such as raft, to stand on an independent discreet spring elements or supports. In 1955, Karl Terzaghi, in his paper '*Evaluation of coefficients of subgrade reaction*' proposed a method to estimate the magnitude of the spring constants. His approach, also known as subgrade reaction model, was then became popular and commonly used in the design of raft foundation.

Looking back into the origin of this concept (see Fig.2), one can see that the modulus or the coefficient of subgrade reaction, $k_s(x)$, is defined as the foundation pressure, p(x), divided by the corresponding settlement of the underlying soil, d(x), i.e.:

$$k_s(x) = \frac{p(x)}{d(x)} \qquad \text{......... (2)}$$

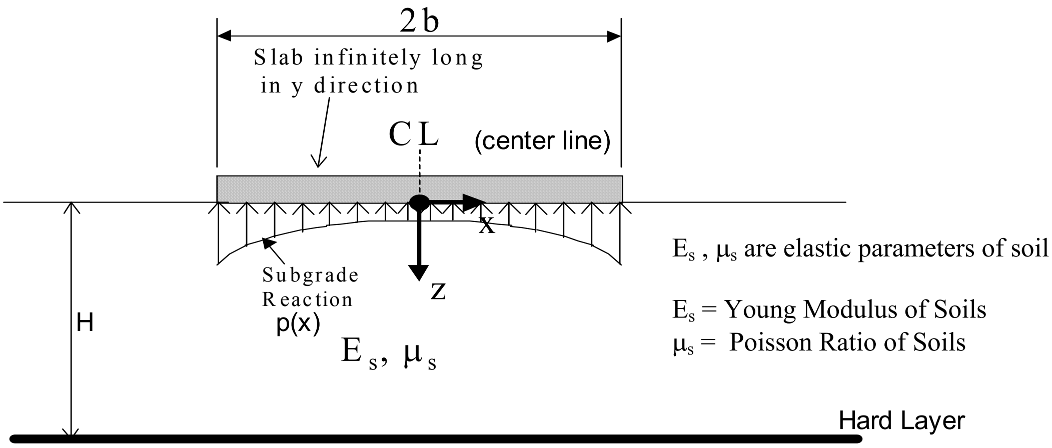

Fig. 2 - Subgrade Reaction under a Flexible Foundation

In other words, the subgrade reaction is no other than the distribution of soil reaction, p(x), beneath the raft foundation structure against the foundation load. The distribution of the soil reaction is not linear in shape. This is particularly true when the foundation is subjected to uniform load. In this case, generally, the distribution of the soil reaction in clayey soils is curving upward, as shown in Fig. 2, with the largest reaction around the edges of the foundation and the smallest reaction around the center. In sandy soils, the reverse reaction is seen, i.e. zero on the edges and maximum at the center point. In principle, the distribution of the soil reactions right beneath the raft foundation depend on the position of the point under consideration (i.e., the distance of x), the shape of the loading and the relative rigidity (EI) of the raft foundation structure against the underlying soils.

The Wrinkler model is a simplified mathematical formulation of an elastic soil model. This concept does not take into account the fact that the foundation reaction or the soil stresses is distributed to the deeper soil layer and forming the so called 'bulb pressure'. The soil settlement beneath the foundation is the accumulation of interactions between the soil stresses and the elastic parameters of the soils at each point inside the bulb pressure zone. Assuming the soils inside the bulb pressure zone posses are homogeneous, Vesic (1961) expanded the Wrinkler model into elastic model and developed the following equation:

$$k_s = \frac{E_s}{B.I_p.\left(1-\upsilon_s^2\right)} \qquad \text{......... (3)}$$

The above Vesic's equation clearly shows that the modulus of subgrade reaction depends not only on the width of the foundation, B, but also on the elastic parameters of soils, E_s and μ_s, and on the shape factor of the foundation, I_p.

In the earlier days, for the sake of mathematical simplicity, it is generally simplified that the spring constant is not a function of the position x (see Fig.2), hence a single value of spring constant is applied. However, the non-linearity distribution of the soil reactions right beneath the foundation structure suggests that the so-called modulus of subgrade reaction, hence the spring constant, is not a unique value. Terzaghi himself recognized the limitation of this assumption. Bowles (1997) suggested providing higher k_s at the edges of the raft and smaller k_s at the center position.

The above explanations show that there is no discrete value of modulus of subgrade reaction for a given type of soil *Therefore, it does not realistic to ask for a spring constant value without the information on the type and the size of the foundation structure.*

In layered soils with different elastic parameters, an equivalent model must be developed in order to derive a representative modulus of subgrade reaction. To do this the elastic settlement of the layered soils induced by the foundation pressure must first be calculated. Poulos and Davis, 1974, mathematical formulation can be used to calculate the elastic settlement of the foundation soils. In a pile raft foundation, to answer the question on the magnitude of the spring constant, the geotechnical engineer also has either to calculate the settlement of the pile foundation or derives it from a pile load test result.

Since the modulus of subgrade reaction (spring constant) is needed to calculate the settlement of the foundation soils, why should one goes to the trouble in providing the spring constant? The structural engineers asked the spring constant because they want to feed in the parameter into their computer software. To the author knowledge, as it is not developed to handle geotechnical problems, the structural engineering software used in analyzing raft or pile raft foundation cannot handle geotechnical parameters.

Another limitation of the spring constant model is the assumption that the foundation soil has linear or elastic behavior. In reality, since Wrinkler introduced his theory (1867) 133 years have lapsed, and the geotechnical engineering has kept on advancing. It has been known that soil behavior does not elastic. It is an elastoplastic material with different behavior within each classification, and many soil models have been developed.

3. PROPER SOIL MODEL AND SOIL STRUCTURE INTERACTION

In order to provide a relatively simple and quick solution for the analysis of raft foundation, Wrinkler followed by Terzaghi, simplified the mathematical formulation into the spring constant or modulus of subgrade reaction model. Over the time, many geotechnical experts had gained better and better understanding on soil behavior and many soil models has been developed. Many of them come with complex mathematical equations, which needs more advanced computer technology and special finite element software to solve.

Until late 1980s where computer hardware, software and run time cost was still very expensive, the spring constant model was indeed one of a good tool for engineers. However, since mid of 1990s and especially as we enter this new millenium, advanced Personal Computer and the relevant geotechnical engineering software has become available and affordable for most firm. So why don't we use a specific finite element method to solve a soil structure interaction problem?

Nowadays, finite element software, such as PLAXIS, CRISP, SIGMA, etc., which is specially developed to solve geotechnical problems has been available. The software, PLAXIS for example, is capable in solving many geotechnical and soil structure interaction problems, such as: diaphragm wall, tunneling, groundwater flow, consolidation, ground anchor and struts, geosynthetic wall, etc. It provides beam element and also slip/interface element, which is very useful in modeling the structural element and the relation of soil-structure interfaces. It also supports various soil models to simulate the behavior of soil continua. A short discussion of the available models is:

- ***Linear elastic model***: This model represents Hooke's law of isotropic linear elasticity. The model involves two elastic stiffness parameters, namely Young's modulus, E, and Poisson's ratio, n. The linear elastic model is very limited for the simulation of soil behavior. It is primarily used for stiff massive structures in the soil.

- ***Mohr-Coulomb model***: This well-known model is used as a first approximation of soil behavior in general. The model involves five parameters, namely Young's modulus, E, Poisson's ratio, n, the cohesion, c, the friction angle, j, and the dilatancy angle, y.

- ***Hardening Soil model***: This is an elastoplastic type of hyperbolic model, formulated in the framework of friction hardening plasticity. This second-order model can be used to simulate the behavior of sands, gravel and overconsolidated clays.

- ***Soft Soil model***: This is a Cam-Clay type model, which can be used to simulate the behavior of soft soils like normally consolidated clays and peat. The model performs best in situations of primary compression.

- ***Soft Soil creep model***: This is a second order model formulated in the framework of viscoplasticity. The model can be used to simulate the time-dependent behavior of soft soils.

It is clear that a suitable soil model can be chosen for a specific problem. The section below presents a case study in solving a soil structure interaction problem with the help of PLAXIS software.

4. A CASE STUDY ON SOIL STRUCTURE INTERACTION

In a densely populated city, it is not uncommon that a subway tunnel must be constructed underneath an existing building foundation or the reverse, that is to construct a building on top of an existing tunnels. In 1998, the author had a chance to evaluate such a problem. At that time a twin tunnel subway project was on its way. These 6.3 m diameter twin tunnels shall cross some 30 m underneath a land where a condominium building was planned. The landlord was wondering when to construct his building, before or after the tunneling?

If the building was constructed before the tunnels passed the area, he had no responsibility on the tunnel construction and it would be the tunnel contractor responsibility to take precaution not to induce any negative impact to the building. However, at that time the macro economy situation was not favorable for the sales of the condominium. On the other hand, if the building was constructed later, the impact of the building construction to the twin tunnels had to be studied. And this might lead to a more costly foundation, as there is a requirement that any pile foundation from the ground surface to the spring-lines of a subway tunnel must not bear any friction resistance. The other option available is to strengthen the tunnel lining to anticipate the future additional stresses that come from the building foundation. And the building owner would have to contribute on its cost.

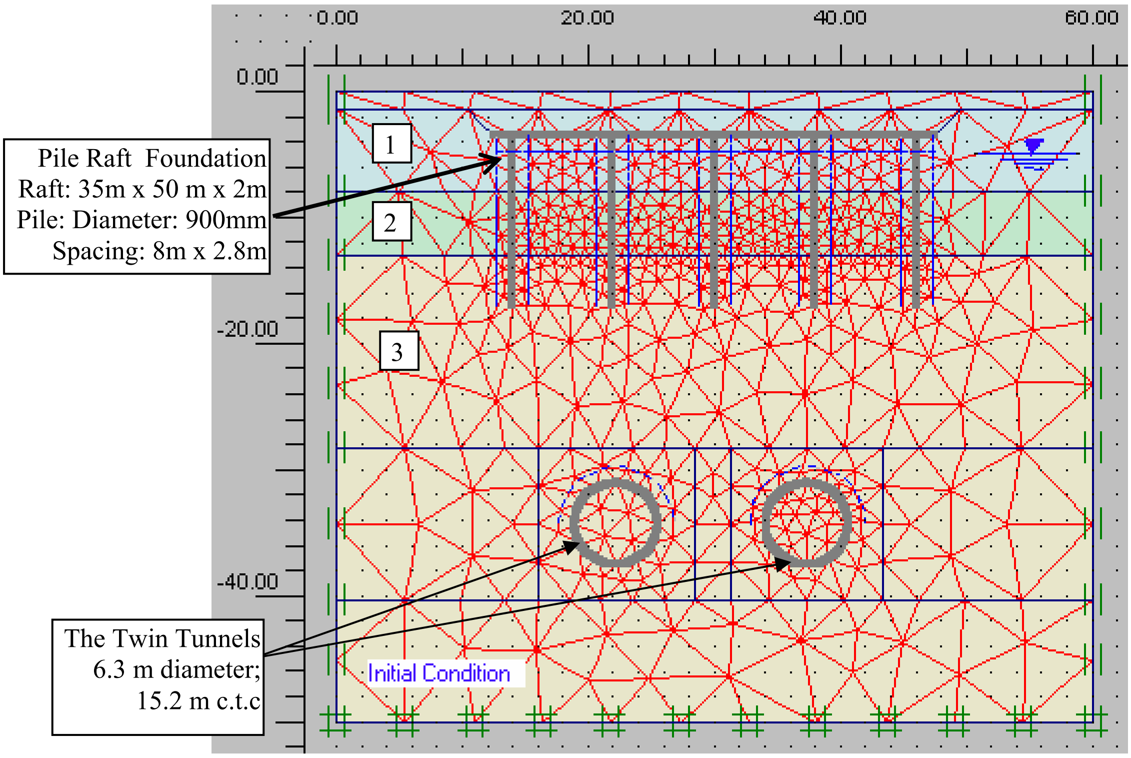

Fig. 3 - The Finite Element Model of The Initial Condition

Figure 3 shows the initial condition of the site and the subsequent soil parameters. The center of the tunnel lines is 35 m below the ground surface. Landscaping of the site required a 1.5 m excavation and this was done before the tunneling. The base of the raft foundation would be around 3.5 m from the ground surface. The groundwater level was found at about 3.75 m below the ground surface. Table 1 shows the soil data. Mohr-Coulomb soil model was adopted to perform the analysis.

Table 1 The Soil Data

Mohr-Coulomb Number	Identification	Type	γ_{dry} [kN/m^3]	γ_{wet} [kN/m^3]	k_x [m/day]	k_y [m/day]	ν [-]	E_{ref} [kN/m^2]
1	stiff silty clay	Drained	16.0	18.0	1.3000E-3	9.0000E-4	0.33	30000.0
2	hard silty clay	Drained	18.0	20.0	1.3000E-3	9.0000E-4	0.33	60000.0
3	sandstone	Drained	19.0	21.0	0.0130	0.0086	0.33	1E5

Number	Identification	c_{ref} [kN/m^2]	φ [°]	ψ [°]	R_{inter} [-]	Interface Permeability [-]
1	stiff silty clay	5.0	22.0	0.0	0.70	Neutral
2	hard silty clay	10.0	30.0	0.0	0.70	Neutral
3	sandstone	15.0	42.0	5.0	0.80	Impermeable

Many possible construction sequences were analyzed. The construction sequence presented in this paper is as follows:

- Overall excavation up to 1.5 m deep.
- Bored piles construction
- Tunneling (followed by volume loss)
- 2.0 m excavation for raft construction
- Raft construction
- Building Construction and Load Application

The results of the final stage construction are presented below,

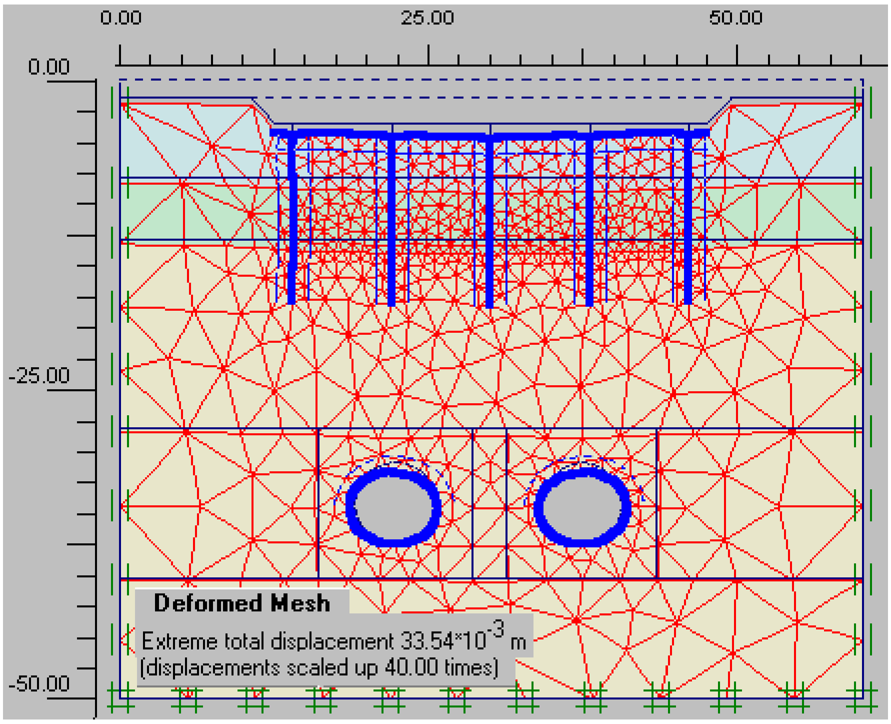

Fig. 4 - Deformed Mesh

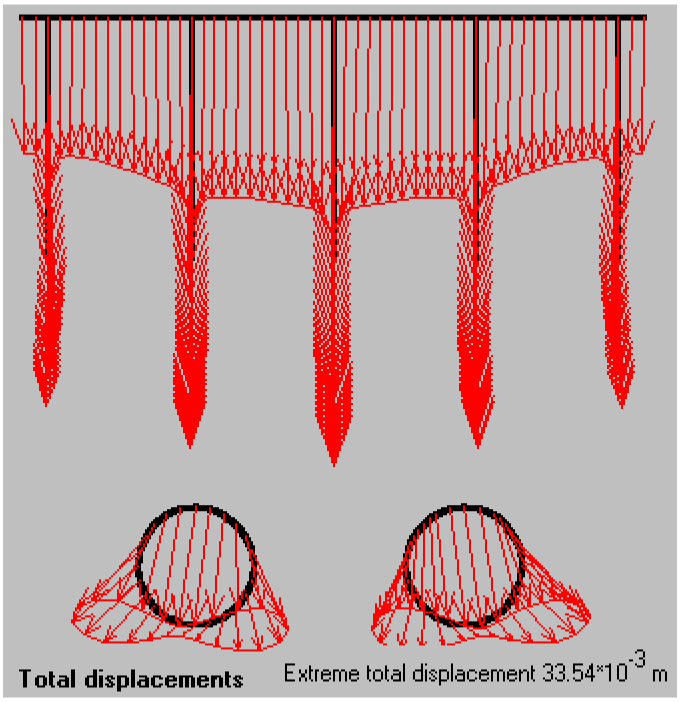

Fig. 5 - Pile Raft and Tunnels Total Displacement

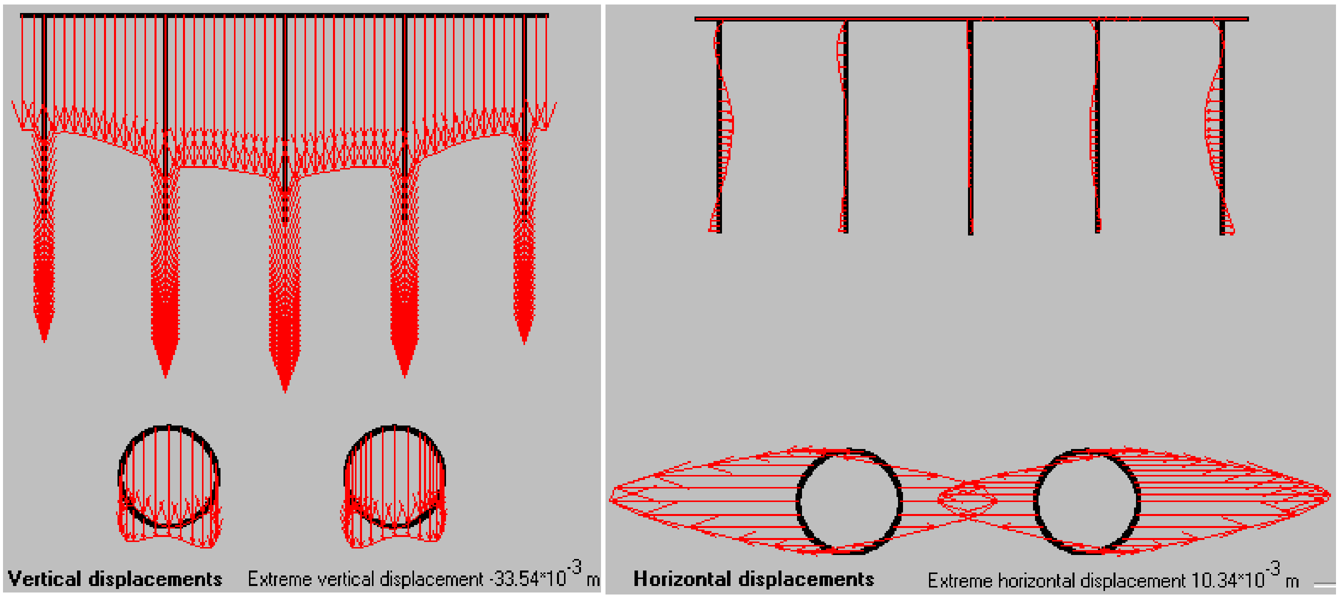

Fig. 6 - Pile Raft and Tunnels Vertical and Horizontal Displacement

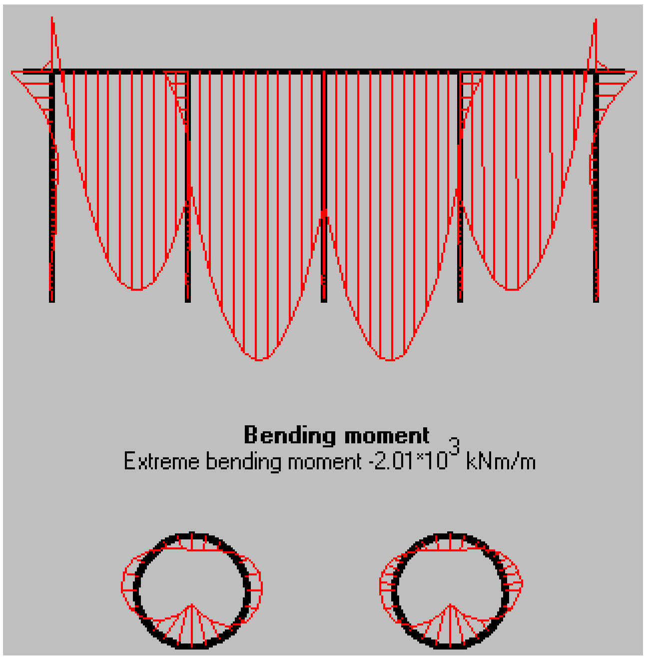

Fig. 7 - Pile Raft and Tunnels Bending Moment

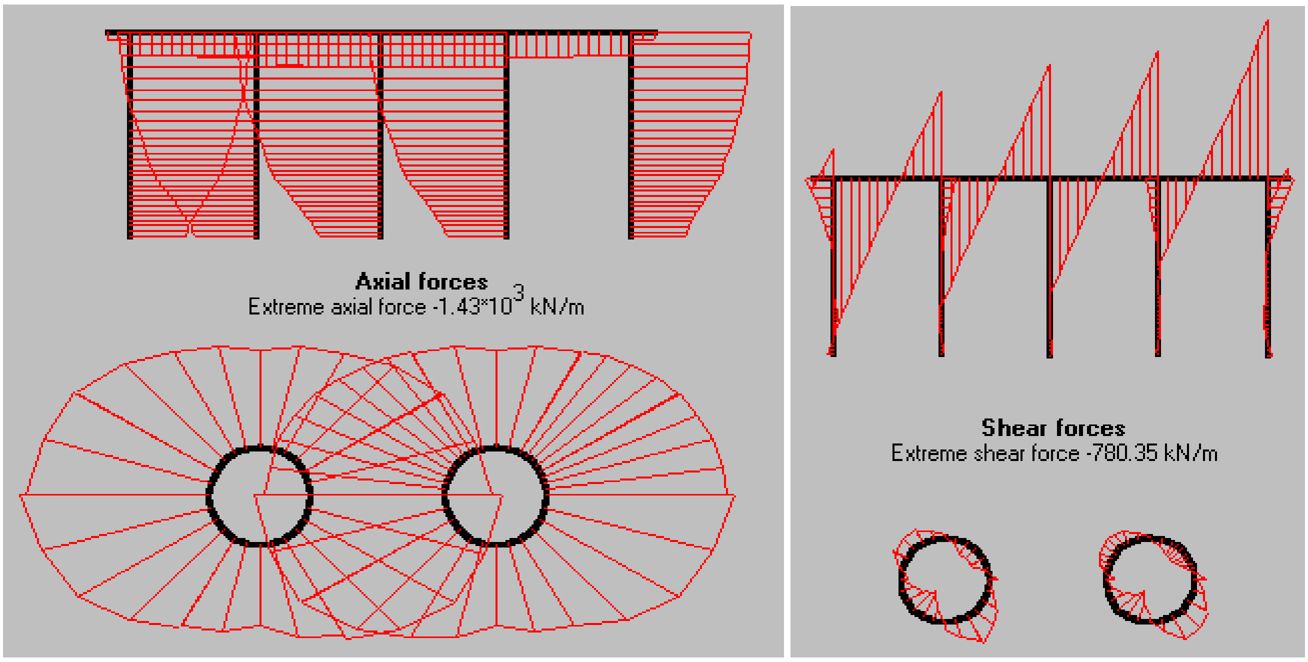

Fig. 8 - Pile Raft and Tunnels Axial and Shear Forces

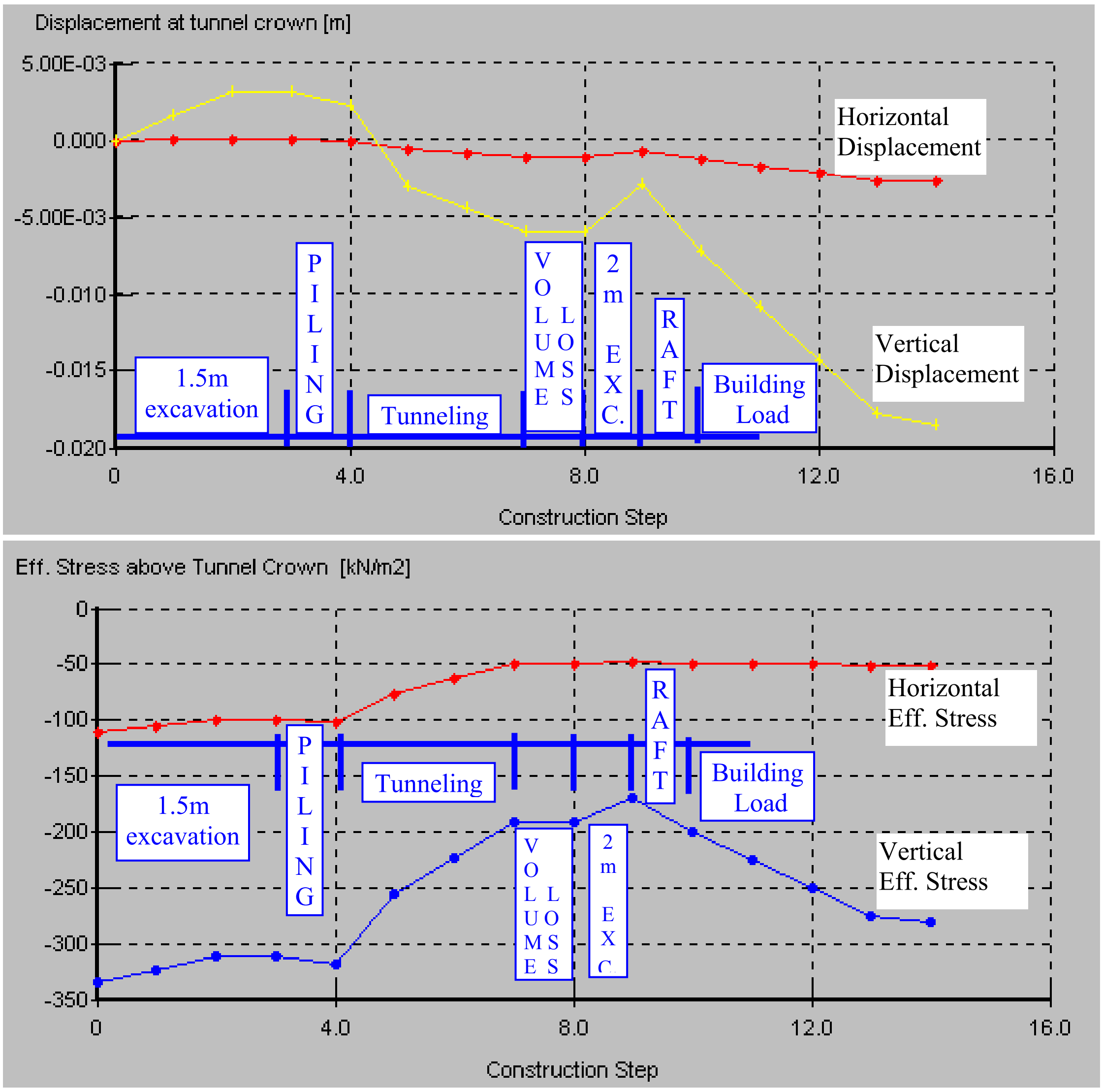

Fig. 9 Changes of Stress and Displacement right above Tunnel Crown

With the said construction sequence, the result of the analysis shows that the maximum pile raft settlement would be in the order of 33 mm. The analysis also predicted that the building would exert additional vertical stress of 90 kN/m^2, with a corresponding 12 mm vertical displacement, to the tunnel crown. The above example was one of the input for project evaluation. It shows the importance of the soil structure interaction analysis, which cannot be solved by using the spring constant model.

5. CLOSURES

The above discussions show that there is no straightforward answer to the question of: "What is the magnitude of the spring constant (or the modulus of subgrade reaction) at this site?" It is inappropriate for a geotechnical engineer to provide the said parameters without knowing the system and the size of the foundation. The non-linearity of soil reaction beneath a footing or raft foundation suggests that the k_s value is not a unique value. Great care must be exercised in deriving the value. It is always important to have a good communication, understanding and cooperation between the structural engineer and the geotechnical engineer in solving a particular foundation problem.

Since the computer technology and the relevant finite element software has become relatively cheap and readily available, whenever possible, it is suggested to perform a soil structure interaction analysis and leave behind the spring constant concept. As demonstrated above, nowadays, the geotechnical finite element software is capable to handle complex soil structure interaction problem, which cannot be solved by the spring constant model. Many soil models have been incorporated into the software. The newest version of PLAXIS software even comes with dynamic module, which is capable to evaluate soil structure interaction due to dynamic load and earthquake loading. Its 3D version is on the final stage of development.

Last but not least, the derivation of the input soil parameters is very important. As soil is not manmade materials, strong theoretical knowledge and sophisticated engineering software alone is not adequate. A geotechnical engineer must gain plenty of practical experiences in order to come out with a sound engineering judgment in determining the relevant soil parameters for a particular soil model. It does not mater how sophisticated computer software is, the adage "Garbage in Garbage out" is always prevails.

REFERENCES

Bowles J.E. (1997), *Foundation Analysis and Design*, 5th ed., McGraw Hill Int.Ed., Singapore.

Brinkgreve R.B.J and Vermeer (1998), *PLAXIS – Finite Element Code for Soil and Rock Analysis*, A.A. Balkema, Rotterdam, The Netherlands.

Poulos H.G. and Davis E.H. (1974), *Elastic Solutions for Soil and Rock Mechanics*, John Willey & Sons, Inc., New York, USA.

Terzaghi K. (1955), Evaluation of coefficients of subgrade reaction, *Geotechnique*, No.4, 297-326.

Terzaghi K., Peck R.B. and Mesri G (1996), *Soil Mechanics in Engineering Practice*, John Willey & Sons, Inc., New York, USA.

Vesic A.B. (1961), Beam on Elastic Subgrade and The Wrinkler's Hypothesis, *Proc. 5th Int. Conf. On Soil Mechanics and Foundation Engineering*, Vol.1, p. 845-850.